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FIXED-AND VARIABLE-TIME-OF-ARRIVAL GUIDANCE MATRICES FOR INTERPLANETARY FLIGHT

By Victor R. Bond and
Thomas B. Murtagh
Advanced Mission Design Branch

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Approved: Jack Funk
Jack Funk, Chief
Advanced Mission Design Branch

Approved: John P. Mayer
John P. Mayer, Chief
Mission Planning and Analysis Division

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SUMMARY

An analytical development of fixed- and variable-time-of-arrival guidance matrices required for the navigation and guidance error analysis of interplanetary missions is presented. The fixed-time-of-arrival guidance matrix is developed subject to the constraint that position vector deviations at the target point are nulled. Two variable-time-of-arrival matrices are derived, the first subject to the constraint that the magnitude of the velocity correction be minimized and the second requiring flight-path angle deviations at the target point be nulled.

INTRODUCTION

The probability of success for any interplanetary mission, either manned or unmanned, is a complicated function of many systems, not the least of which are the spacecraft navigation and guidance systems. The navigation system processes onboard observations of planets and/or stars with appropriate sensors and provides the guidance system with a "best" estimate of the vehicle's state with respect to some primary body, usually the earth. Ground-based radar tracking also provides the same information and is generally superior to the onboard system except during certain unfavorable spacecraft/radar geometries. The guidance system then commands a maneuver to correct the spacecraft trajectory such that specified mission constraints are satisfied.

For a given navigation system, the statistical analysis of a guided flight depends to a large extent on the type of targeting required, and this, in turn, determines the guidance law to be implemented. Linearized impulsive guidance techniques reduce the complexity involved in formulating the guidance law and in developing the guidance matrix and, in general, do not involve unreasonable assumptions (ref. 1).

The purpose of this note is to develop a generalized expression for the impulsive velocity correction in terms of the state deviations just prior to the correction and the position deviation at the target point after the correction. The fixed- and variable-time-of-arrival guidance matrices are then derived from this general expression. The fixed-time-of-arrival matrix is developed subject to the constraint that position vector deviations at the target point are zeroed. Two variable-time-of-arrival matrices are derived, the first requiring that the magnitude of the velocity correction be minimized and the second nulling flight-path angle deviations at the target point.

An application of these guidance matrices to the navigation and guidance analysis of a Mars probe launched from a manned flyby spacecraft is presented in reference 2.

ANALYSIS

Guidance System Error Equations

In order to perform a complete statistical evaluation of a guided flight, it is necessary to propagate and update not only the covariance matrix of the state uncertainties, $E(t)$, but also the covariance matrix of the state dispersions, $X(t)$. Both matrices are propagated using the equations

$$E(t) = \Phi(t,t_o)E(t_o)\Phi^T(t,t_o) \tag{1}$$

and

$$X(t) = \Phi(t,t_o)X(t_o)\Phi^T(t,t_o) \tag{2}$$

where $\Phi(t, t_o)$ is the state transition matrix evaluated between t_o and t, and can be partitioned into four 3 x 3 sub matrices as follows

$$\Phi(t, t_o) = \begin{bmatrix} \phi_{11}(t, t_o) & \phi_{12}(t, t_o) \\ \phi_{21}(t, t_o) & \phi_{22}(t, t_o) \end{bmatrix} \tag{3}$$

If a navigation measurement is made at time t, the matrix X(t) remains unchanged. However, if a guidance correction is commanded, then both E(t) and X(t) are updated according to the following equations (ref. 3):

$$E(t)^{+} = E(t)^{-} + MN(t)M^{T} \tag{4}$$

$$X(t)^{+} = \left[I + G(t)\right]\left[X(t)^{-} - E(t)^{-}\right]\left[I + G(t)\right]^{T} + E(t)^{+} \tag{5}$$

where $(\ )^{+}$ indicates the matrix after the correction and $(\ )^{-}$ represents the matrix immediately prior to the correction. The 6 x 3 matrix M is defined by

$$M = \begin{pmatrix} 0 \\ I \end{pmatrix} \tag{6}$$

where I is a 3 x 3 identity matrix. The matrix N(t) is defined as the covariance matrix of the velocity correction uncertainty and is derived and discussed in reference 4. The 6 x 6 matrix G(t) is referred to as the guidance matrix and is a function of the type of guidance law formulated (ref. 2). This matrix also may be partitioned into four 3 x 3 submatrices as

$$G(t) = \begin{bmatrix} 0 & 0 \\ G_1(t) & G_2(t) \end{bmatrix} . \tag{7}$$

The derivation and discussion of this matrix is presented in the sections which follow.

Fixed Time of Arrival (FTA) and Variable Time of Arrival (VTA) Guidance

A small velocity correction is to be made at time t to a trajectory to guarantee that certain terminal conditions will be met. There are two general classes of terminal constraints which may be considered: fixed-time-of-arrival (FTA) constraints and variable-time-of-arrival constraints (VTA). This analysis is essentially an interpretation of FTA and VTA guidance presented in reference 5, but obtains the final results in terms of the familiar state transition matrix.

At the instant before the velocity correction, the terminal position $\underline{r}_T^-$ is related to the position and velocity just prior to correction time by

$$\underline{r}_T^- = \underline{r}_T^-(\underline{r}^-, \underline{v}^-). \tag{8}$$

Lack of a subscript on $\underline{r}$ and $\underline{v}$ means they are evaluated at correction time t; a subscript (T) means they are evaluated at the terminal time, T.

Taking linear deviations of equation (8),

$$\delta \underline{r}_T^- = \frac{\partial \underline{r}_T^-}{\partial \underline{r}^-}\delta \underline{r}^- + \frac{\partial \underline{r}_T^-}{\partial \underline{v}^-}\delta \underline{v}^- . \tag{9}$$

At the instant after the velocity correction, the terminal position $\underline{r}_T^+$ is related to the position and velocity just after correction time by

$$\underline{r}_T^+ = \underline{r}_T^+(\underline{r}^+, \underline{v}^+) . \tag{10}$$

Taking linear deviations,

$$\delta \underline{r}_T^+ = \frac{\partial \underline{r}_T^+}{\partial \underline{r}^+}\delta \underline{r}^+ + \frac{\partial \underline{r}_T^+}{\partial \underline{v}^+}\delta \underline{v}^+ . \tag{11}$$

Since there can be no instantaneous change in position,

$$\underline{r}^+ = \underline{r}^- = \underline{r} . \tag{12}$$

and

$$\delta \underline{r}^+ = \delta \underline{r}^- = \delta \underline{r} . \tag{13}$$

If the correction is assumed small and the trajectories are linearly related,

$$\frac{\partial \underline{r}_T^-}{\partial \underline{r}^-} = \frac{\partial \underline{r}_T^+}{\partial \underline{r}^-} \tag{14}$$

$$\frac{\partial \underline{r}_T^-}{\partial \underline{v}^-} = \frac{\partial \underline{r}_T^+}{\partial \underline{v}^-} . \tag{15}$$

These partials may be recognized immediately as being the submatrices $\phi_{11}(T,t)$ and $\phi_{12}(T,t)$ of the state transition matrix. (For the conic case these are given in reference 6.) Using these simplifications, equations (9) and (11) may be written as

$$\delta \underline{r}_T^- = \phi_{11}\delta\underline{r} + \phi_{12}\delta\underline{v}^- \tag{16}$$

$$\delta \underline{r}_T^+ = \phi_{11}\delta\underline{r} + \phi_{12}\delta\underline{v}^+ \tag{17}$$

where $\phi_{11} = \phi_{11}(T,t)$, $\phi_{12} = \phi_{12}(T,t)$.
To first order $\delta\underline{v}^-$ and $\delta\underline{v}^+$ are related to the nominal trajectory by

$$\delta\underline{v}^- = \underline{v}^- - \underline{v}_{NOM} \tag{18}$$

$$\delta\underline{v}^+ = \underline{v}^+ - \underline{v}_{NOM}\ . \tag{19}$$

So the velocity correction $\Delta\underline{v}$ is found from

$$\Delta\underline{v} = \underline{v}^+ - \underline{v}^-$$

or

$$\Delta\underline{v} = \delta\underline{v}^+ - \delta\underline{v}^-\ . \tag{20}$$

Now solve (16) for $\delta\underline{v}^-$ and (17) for $\delta\underline{v}^+$ and difference them to obtain $\Delta\underline{v}$.

$$\Delta\underline{v} = \phi_{12}^{-1}\left[\delta\underline{r}_T^+ - \delta\underline{r}_T^-\right]\ .$$

Replacing $\delta\underline{r}_T^-$ by (16), the $\Delta\underline{v}$ becomes

$$\Delta\underline{v} = \phi_{12}^{-1}\left[\delta\underline{r}_T^+ - \phi_{11}\delta\underline{r} - \phi_{12}\delta\underline{v}^-\right]\ . \tag{21}$$

This equation expresses the linear impulsive guidance law in a very general form. Its application is made by constraining up to three terminal conditions. Some of these applications are discussed in the following sections.

FTA Guidance.- In FTA guidance, it is desired to null the position deviations $\delta\underline{r}_T^+$ at the fixed terminal time T, that is,

$$\delta\underline{r}_T^+ = 0 \quad . \tag{22}$$

The $\Delta\underline{v}$ for FTA, $\Delta\underline{v}$ becomes

$$\Delta\underline{v}_F = -\phi_{12}^{-1}\phi_{11}\delta\underline{r} - \delta\underline{v}^-$$

or

$$\Delta\underline{v}_F = G_1\delta\underline{r} + G_2\delta\underline{v} \tag{23}$$

where

$$G_1 = -\phi_{12}^{-1}\phi_{11},\ G_2 = -I \quad . \tag{24}$$

VTA Guidance.- In this type of guidance, the terminal time T is allowed to be free. As an example, let the component of the position deviation that is parallel to the velocity at the terminal time be free.

$$\delta\underline{r}_T^+ = -\underline{v}_T\delta T \quad . \tag{25}$$

Substituting into (21), the $\Delta\underline{v}$ for VTA, $\Delta\underline{v}_V$, is

$$\Delta\underline{v}_V = \phi_{12}^{-1}\left[-\underline{v}_T\delta T - \phi_{11}\delta\underline{r} - \phi_{12}\delta\underline{v}^-\right] \quad . \tag{26}$$

The VTA correction can then be expressed in terms of the FTA correction or

$$\Delta\underline{v}_V = \Delta\underline{v}_F - \phi_{12}^{-1}\underline{v}_T\delta T \quad . \tag{27}$$

By defining

$$\underline{W} = \phi_{12}^{-1}\underline{v}_T \tag{28}$$

the equation (27) becomes

$$\Delta\underline{v}_V = \Delta\underline{v}_F - \underline{W}\delta T \quad . \tag{29}$$

Since there is an additional degree of freedom remaining, δT may be chosen, if desired, to mimimize $|\Delta \underline{v}_V|$.

From (23),

$$(\Delta v_V)^2 = (\Delta v_F)^2 - 2\underline{W} \cdot \Delta \underline{v}_F \delta T + \underline{W} \cdot \underline{W} \delta T^2$$

$$\frac{\partial (\Delta v_V)^2}{\partial (\delta T)} = -2\underline{W} \cdot \Delta \underline{v}_F + 2\underline{W} \cdot \underline{W} \delta T = 0$$

or

$$\delta T = - \frac{\underline{W} \cdot \Delta \underline{v}_F}{\underline{W} \cdot \underline{W}} \quad . \tag{30}$$

Substituting back into (29),

$$\Delta \underline{v}_V = \left(I - \frac{\underline{W}\,\underline{W}^T}{\underline{W} \cdot \underline{W}} \right) \Delta \underline{v}_F = G_1 \delta \underline{r} + G_2 \delta \underline{v}^- \tag{31}$$

where

$$G_1 = - \left(I - \frac{\underline{W}\,\underline{W}^T}{\underline{W} \cdot \underline{W}} \right) \phi_{12}^{-1} \phi_{11}$$

$$G_2 = - \left(I - \frac{\underline{W}\,\underline{W}^T}{\underline{W} \cdot \underline{W}} \right) \tag{32}$$

<u>VTA Guidance, Null $\delta\gamma_T$</u>.- The extra degree of freedom in VTA guidance may also be used to control an additional terminal deviation if desired. As an example, the errors in flight-path angle at the terminal time may be nulled.

The deviation in flight-path angle, $\delta\gamma$, is shown in reference 7 to be a function of position and velocity deviations at terminal time and is defined as

$$\delta\gamma_T = \underline{z}_1^T \delta \underline{r}_T^+ + \underline{z}_2^T \delta \underline{v}_T^+ \ , \tag{33}$$

but $$\delta \underline{r}_T^+ = \phi_{11}\delta \underline{r} + \phi_{12}\delta \underline{v}^+ \tag{17}$$

and
$$\delta \underline{v}_T^+ = \phi_{21}\delta \underline{r} + \phi_{22}\delta \underline{v}^+ \quad . \tag{34}$$

Solve these two equations for $\delta \underline{v}_T^+$, eliminating $\delta \underline{v}^+$:

$$\delta \underline{v}_T^+ = \phi_{21}\delta \underline{r} + \phi_{22}\phi_{12}^{-1}\left[\delta \underline{r}_T^+ - \phi_{11}\delta \underline{r}\right]$$

or

$$\delta \underline{v}_T^+ = \left[\phi_{21} - \phi_{22}\phi_{12}^{-1}\phi_{11}\right] \delta \underline{r} + \phi_{22}\phi_{12}^{-1}\delta \underline{r}_T^+ \quad . \tag{35}$$

Now substitute this equation into equation (33) to eliminate $\delta \underline{v}_T^+$.

$$\delta \gamma_T = \left[\underline{z}_1^T + \underline{z}_2^T\phi_{22}\phi_{12}^{-1}\right]\delta \underline{r}_T^+ + \underline{z}_2^T\left[\phi_{21} - \phi_{22}\phi_{12}^{-1}\phi_{11}\right]\delta \underline{r} \quad . \tag{36}$$

Using (25), set $\delta \gamma_T = 0$ and solve for δT.

$$\delta T = \frac{\underline{z}_2^T\left[\phi_{21} - \phi_{22}\phi_{12}^{-1}\phi_{11}\right]\delta \underline{r}}{\left[\underline{z}_1^T + \underline{z}_2^T\phi_{22}\phi_{12}^{-1}\right]\underline{v}_T} \quad . \tag{37}$$

This equation is the time deviation required for the deviation in flight-path angle to be nulled. Substituting (37) into (26),

$$(\Delta \underline{v}_v)_{\delta \gamma_T = 0} = G_1\delta \underline{r} + G_2\delta \underline{v}^- \tag{38}$$

where
$$G_1 = -\frac{\phi_{12}^{-1}\underline{v}_T\underline{z}_2^T\left[\phi_{21} - \phi_{22}\phi_{12}^{-1}\phi_{11}\right]}{\left[\underline{z}_1^T + \underline{z}_2^T\phi_{22}\phi_{12}^{-1}\right]\underline{v}_T} - \phi_{12}^{-1}\phi_{11}$$

$$G_2 = -I \quad . \tag{39}$$

These guidance laws may be applied directly to the guidance-to-entry problem discussed in references 2 and 7. The terminal conditions subscripted by (T) are the same as the entry conditions subscripted by (E).

CONCLUDING REMARKS

A general linear impulsive guidance law has been derived for correcting spacecraft trajectories in a gravitational field by small impulsive velocity corrections. Fixed-time-of-arrival and variable-time-of-arrival guidance matrices are then developed by considering the terminal constraints that modify the general guidance law. The fixed-time-of-arrival guidance law is derived by constraining all three position deviations at the terminal time. The variable-time-of-arrival guidance law is derived for the case of one free terminal position deviation component that is parallel to the terminal velocity, allowing a degree of freedom which may be used to minimize the velocity correction or to null the errors in terminal flight-path angle.

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